On $u_{n+2} = \frac{\sqrt{2}}{n+2} - \frac{n+1}{n+2} u_n$ ($u_{n+2} \to l$, $\frac{\sqrt{2}}{n+2} \to 0$, $\frac{n+1}{n+2} \to 1$, $u_n \to l$)

par suite

$$\lim_{n \to +\infty} \frac{\sqrt{2}}{n+2} = 0$$

et

$$\lim_{n \to +\infty} \frac{n+1}{n+2} = \lim_{n \to +\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(1+\frac{2}{n}\right)}$$

$$= \lim_{n \to +\infty} \frac{1+\frac{1}{n}}{1+\frac{2}{n}} = 1$$

($1+\frac{1}{n} \to 1+0$, $1+\frac{2}{n} \to 1+0$)

$\Rightarrow l = 0 - l$

$\Rightarrow 2l = 0$

$\Rightarrow l = 0$

Ainsi, $\lim_{n \to +\infty} u_n = 0$